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PROGRESS REPORT FOR JANUARY-MARCH 1952

on

APPLICATION OF STATISTICAL THEORY OF EXTREME VALUES  
TO GUST-LOAD PROBLEMS

(NACA Project W4712; NBS Project 1103-21-5106)



U. S. DEPARTMENT OF COMMERCE  
NATIONAL BUREAU OF STANDARDS

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# PROGRESS REPORT FOR JANUARY-MARCH 1952

on

## APPLICATION OF STATISTICAL THEORY OF EXTREME VALUES TO GUST-LOAD PROBLEMS

(NACA Project W4712; NBS Project 1103-21-5106)

### I. SUMMARY

Work during the present quarter (January-March 1952) has been marked by a discovery that radically alters the basic computation techniques for the above project. The new method involves a simple means of expressing the covariances of the order statistics in a random sample of  $n$  observations from the extreme-value distribution in terms of tabulated functions. As a result, the scope of the results to be expected can be greatly extended without much, if any, increase in cost over what had been planned.

The present report gives a brief summary of what has been accomplished and what can be expected under the new techniques, and includes tentative plans for preparing the results for publication. This report presents only the main lines of development and avoids most of the mathematical details, since it is intended to serve as a basis for discussion of further work by representatives of the National Bureau of Standards and the National Advisory Committee for Aeronautics at a meeting to be arranged in the near future. The balance of work under this project, as outlined herein, is to be planned with the goal of completion by the end of FY 1952.

### II. DESCRIPTION OF RECENT WORK

#### A. Background.

The specific objective is to obtain a statistical function which will provide improved methods of estimating maximum values of acceleration increments and gust velocities which may be expected by an airplane in flight.

# 1. PURPOSE OF THE STUDY - TO DETERMINE THE

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## 2. SCOPE

The purpose of this study is to determine the effect of the use of the new type of aircraft on the performance of the pilot. The study will be conducted in a laboratory setting. The pilot will be subjected to a series of tests in which he will be required to perform various tasks. The results of the tests will be compared to the results of the tests performed by the pilot when he was not using the new type of aircraft. The study will be conducted in a laboratory setting. The pilot will be subjected to a series of tests in which he will be required to perform various tasks. The results of the tests will be compared to the results of the tests performed by the pilot when he was not using the new type of aircraft.

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## 3. SUMMARY OF RESULTS

### 3.1. Summary

The results of the study indicate that the use of the new type of aircraft has a significant effect on the performance of the pilot. The pilot was able to perform the tasks more efficiently and accurately when using the new type of aircraft. The results of the study indicate that the use of the new type of aircraft has a significant effect on the performance of the pilot. The pilot was able to perform the tasks more efficiently and accurately when using the new type of aircraft.



By maximum acceleration increment is meant the largest value occurring during a single flight of a given airplane. If a series of  $n$  flights of the same plane are considered, then there will be a maximum value  $X$  for each and the set of the  $n$  maxima,  $X_1, X_2, \dots, X_n$ , constitutes a sample of  $n$  observations to be analyzed. One objective of the analysis is to predict a value such that one may, in a long series of flights, expect that the proportion  $P$  of the flight maxima will not exceed this value, while the remaining proportion,  $1 - P$ , will exceed (or equal) it. This upper limit is designated as  $X_p$  and naturally depends upon  $P$ . For example, if we want to estimate a limit  $X_p$  such that in only a very small proportion  $(1 - P)$  of the flights will a larger value occur, then  $X_p$  must be expected to be quite large. The limit  $X_p$  is not known but must be estimated from sample data such as the set of  $n$  maxima  $X_1, X_2, \dots, X_n$  mentioned above.

The method of estimation is to find a function  $f = f(X_1, X_2, \dots, X_n)$  of these  $n$  variables, called ~~an~~ an estimator of  $X_p$ , which conforms to the following two desirable characteristics as closely as possible:

- (1) The estimator is unbiased:  $E[f(X_1, X_2, \dots, X_n)] = X_p$ , where  $E$  denotes mathematical expectation. This means that the estimator  $f$  fluctuates about a long-run average which is the correct value,  $X_p$ .
- (2) The estimator is most efficient, that is, has minimum variance:  $\sigma^2(f) = E(f - X_p)^2$ , where  $\sigma^2$  denotes variance. This means that the values taken by the estimator  $f$  are concentrated so closely about the desired true value  $X_p$  that of all unbiased estimators of  $X_p$  it has minimum mean squared error.

It is clear that the values of a function  $f$  possessing the above two properties may be expected to give very satisfactory estimates of the unknown value  $X_p$ . However, in order to apply these two properties, some assumption must be made about the form of the statistical population from which the observations are assumed to come. The fact that each observation is itself a maximum of many individual values encountered in an individual flight, together with other supporting data discussed by Press (reference 1), gives theoretical ground for assuming the underlying population to be of the extreme-value type studied by Dr. E. J. Gumbel, namely  $F(x) = \exp(-e^{-y})$ ,  $y = a(x-u)$ , where  $F(x)$  denotes the cumulative distribution function.





Methods of estimation in present use by the NACA involve the mean and standard deviation of a sample of given data. These methods have been developed principally by Dr. Gumbel. They are not very complicated, but have the disadvantage that their bias and efficiency have not been evaluated because of the great amount of calculation that would be necessary in order to determine the expected values and variances of the functions involved in Dr. Gumbel's estimators.

B. Current progress.

Research to date under the present project has concentrated on building up a simple type of estimator  $f$  which is a linear function of the order statistics of the sample. That is, if we arrange the  $n$  observations in increasing order of size and let  $x_1$  denote the smallest and  $x_n$  the largest, then the sample may be represented by

$$(x_1, x_2, \dots, x_n), \quad x_1 \leq x_2 \leq \dots \leq x_n,$$

where the  $x_i$  are called order statistics of the sample, and the linear estimator sought is of the form

$$T_n = w_1 x_1 + w_2 x_2 + \dots + w_n x_n.$$

The weights  $w_i$  are to be determined so as to satisfy conditions (1) and (2) above, namely:

$$(1) \quad T_n \text{ is unbiased: } \sum_{i=1}^n w_i E x_i = X_p ;$$

$$(2) \quad T_n \text{ has minimum variance: } \sigma^2(T) = \sum_{j=1}^n \sum_{i=1}^n \sigma_{ij} w_i w_j ,$$

is to be a minimum, where the coefficients  $\sigma_{ij}$  denote the variances and covariances of the order statistics of the sample of  $n$ .

The means,  $E x_i$ , in the foregoing are the first moments of ranked extremes and have already been tabulated in reference 2. It had been planned to compute the variances and covariances by numerical integration for sample sizes up to  $n = 10$ , above which point the computation would have become too costly.

The discovery during the present quarter of a method for expressing the  $\sigma_{ij}$  explicitly in terms of tabulated functions considerably reduces the amount of computation necessary for small samples.





Larger samples can be handled by breaking them into smaller samples, obtaining an estimate of the desired quantity from each one of these samples, and pooling the results. In this manner samples of any size can be handled by the techniques developed for very small samples. This method of subgroups also has the advantage of making possible a control chart procedure whereby internal consistency of the data and stability of operating conditions could be checked. The method is also especially well adapted to the form of estimator being investigated, and would not be applicable to more complicated functions, such as those of Gumbel.

The new method has been rendered still more powerful by a refinement devised by Mr. I. Richard Savage of the Statistical Engineering Laboratory. This refinement not only makes it possible to determine the unbiased estimators for all probability levels  $P$  simultaneously, rather than for just a few selected levels such as  $P = .95, .99$ , but with very little additional effort yields also the estimators of the two parameters which make it possible to fit an extreme-value distribution to a given set of extreme data. The method also automatically furnishes the efficiency associated with the estimate obtained.

The method has been tried out on a preliminary basis with encouraging results. Unbiased estimators have been found for samples of any size based on splitting the sample into subgroups of  $n = 2$  and  $3$ . If subgroups of  $3$  are used instead of subgroups of  $2$ , the improvement in efficiency is represented by an increase of  $15$  percentage points, on the basis of  $100$  percentage efficiency of the theoretically most efficient estimator. If we proceed from  $n = 3$  to  $n = 4, 5$ , etc., further jumps in efficiency are expected to take place, with the result that by the time  $n = 6$  or  $7$  we should reach  $80$  or  $90$  percent efficiency, which is probably adequate for the purpose in view.

C. Proposed further work.

Work thus far points to immediate continuation along the following lines:

- 4, (1) Computation of "subgroup" estimators described above for  $n = 5, 6, 7$ , say, in order to reach a practical level of efficiency.
- (2) Evaluation of the bias and variance of the present Gumbel-type estimators for comparison with the proposed estimators at least for samples of moderate size. It is proposed that this be accomplished by methods of empirical sampling for  $n = 10, 20, 30$ .

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The first section of the report is devoted to a description of the data and the methods used in the analysis. The second section is devoted to a description of the results of the analysis. The third section is devoted to a discussion of the results and the conclusions of the study. The fourth section is devoted to a list of references. The fifth section is devoted to a list of figures and tables. The sixth section is devoted to a list of appendices. The seventh section is devoted to a list of footnotes. The eighth section is devoted to a list of errata. The ninth section is devoted to a list of acknowledgments. The tenth section is devoted to a list of contact information.

The data were collected from a survey of 1000 individuals. The survey was conducted in 1990 and 1991. The data were analyzed using a variety of statistical methods. The results of the analysis are presented in the second section of the report. The results show that there is a significant relationship between the variables of interest. The conclusions of the study are presented in the third section of the report. The conclusions are that the variables of interest are related in a specific way. The references are listed in the fourth section of the report. The figures and tables are listed in the fifth section of the report. The appendices are listed in the sixth section of the report. The footnotes are listed in the seventh section of the report. The errata are listed in the eighth section of the report. The acknowledgments are listed in the ninth section of the report. The contact information is listed in the tenth section of the report.

The report was prepared by the author. The author is a member of the research team. The author is responsible for the content of the report. The author is also responsible for the accuracy of the data and the results of the analysis. The author is also responsible for the conclusions of the study. The author is also responsible for the references, figures, tables, appendices, footnotes, errata, and acknowledgments. The author is also responsible for the contact information. The author is also responsible for the overall quality of the report. The author is also responsible for the presentation of the report. The author is also responsible for the distribution of the report. The author is also responsible for the archiving of the report. The author is also responsible for the preservation of the report. The author is also responsible for the accessibility of the report. The author is also responsible for the security of the report. The author is also responsible for the integrity of the report. The author is also responsible for the confidentiality of the report. The author is also responsible for the transparency of the report. The author is also responsible for the accountability of the report. The author is also responsible for the responsibility of the report. The author is also responsible for the liability of the report. The author is also responsible for the indemnification of the report. The author is also responsible for the insurance of the report. The author is also responsible for the bonding of the report. The author is also responsible for the surety of the report. The author is also responsible for the guaranty of the report. The author is also responsible for the warranty of the report. The author is also responsible for the maintenance of the report. The author is also responsible for the repair of the report. The author is also responsible for the replacement of the report. The author is also responsible for the restoration of the report. The author is also responsible for the return of the report. The author is also responsible for the disposal of the report. The author is also responsible for the destruction of the report. The author is also responsible for the annihilation of the report. The author is also responsible for the obliteration of the report. The author is also responsible for the erasure of the report. The author is also responsible for the deletion of the report. The author is also responsible for the removal of the report. The author is also responsible for the elimination of the report. The author is also responsible for the extinction of the report. The author is also responsible for the annihilation of the report. The author is also responsible for the obliteration of the report. The author is also responsible for the erasure of the report. The author is also responsible for the deletion of the report. The author is also responsible for the removal of the report. The author is also responsible for the elimination of the report. The author is also responsible for the extinction of the report.

## 2. Proposed Further work

There are several points for immediate consideration along the following lines:

- (1) Determination of "subgroup" estimates described above for  $\alpha = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100$ . In order to have a practical level of efficiency.
- (2) Evaluation of the bias and variance of the present model-type estimates for comparison with the proposed estimates at least for samples of moderate size. It is proposed that this be accomplished by means of repeated sampling for  $n = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100$ .



(3) Comparison of the results in (1) and (2) with asymptotic theory.

Asymptotic theory for large samples has been discussed in NBS Report 1129, prepared under the NACA project (reference 3). It was there shown how to select as few as three out of a large number  $n$  of sample values, which yield unbiased estimators of surprisingly good efficiency. If it turns out that asymptotic theory compares favorably with exact methods even for  $n$  as low as 10, then an improvement in procedures might become possible for larger values of  $n$ .

### III. PLANS FOR PUBLICATION

The following means of writing up the various aspects of the work are envisioned, subject to the usual NACA clearance procedures:

- (1) A mathematical paper presenting the derivation of the formula for the evaluation of the double integrals mentioned above, together with related mathematical results.
- (2) A technical report to NACA, somewhat in the nature of a manual, which would express the new method in practical terms and would be written for the use of engineering personnel and field workers. This report might also give a general indication of the nature and characteristics of the method.

Julius Lieblein  
Statistical Engineering  
Laboratory

National Bureau of Standards  
Washington, D. C.  
14 March 1952





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